Optimal Conjunctive Use of Groundwater and Recycled Wastewater

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Abstract:

Inasmuch as water demand is multifaceted, infrastructure planning should be part of a general specification of efficient quantities and qualities of water deliveries over time. Accordingly, we develop a two-sector dynamic optimization model to solve for the optimal trajectories of groundwater extraction and water recycling. For the case of spatially increasing costs, recycled water serves as an intermediate resource in transition to the desalination steady state. For constant unit recycling cost, recycled wastewater eventually supplies non-potable users as a sector-specific backstop, while desalination supplements household groundwater in the steady state. In both cases, recycling water increases welfare by shifting demand away from the aquifer, thus delaying implementation of costly desalination. Implementation of the model provides guidance on the appropriate timing and size of backstop and recycling infrastructure as well as water deliveries from the various sources to the water-demand sectors.

Keywords: Renewable resources, dynamic optimization, groundwater allocation, wastewater reuse, recycling, reclamation, water quality

JEL codes: Q25, Q28, C6

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1 Introduction

Water scarcity has long been an important issue in many regions around the world, and the threat of climate change has recently brought it even further to the forefront of policy discussions. The United Nations (2006) recommends a multidisciplinary approach to managing water scarcity, inasmuch as "water scarcity affects all social and economic sectors and threatens the sustainability of the natural resources base." While demand for water continues to grow, a plethora of both demand- and supply-side management strategies are being considered, including but not limited to expansion of existing reservoirs or creation of new ones, watershed conservation, more efficient conjunctive use of ground and surface water, new pricing structures, voluntary or mandatory quantity restrictions, and implementation of wastewater recycling and desalination.

The recycling of urban wastewater refers to the process of using treated wastewater for various purposes, including artificial recharge of groundwater basins, irrigation for landscaping or agriculture, and industrial processes that do not require potable water such as cooling. Wastewater can be treated to varying degrees and the resulting level of quality ultimately constrains the recycled water to particular end-uses.

Table 1. Adopted from Abu-Zeid (1998)

Treatment:	Preliminary	Primary	Secondary	Tertiary
What gets removed:	Large solids	Settleable solids by sedimentation	Organic matter	Nitrogen, phosphorus, detergents, softeners, and heavy metals
Agricultural uses:	Non-consumed crops (e.g. wood trees)	Non-consumed crops (e.g. cotton) or non-leafy crops (e.g. orchards)	Animal food crops, food crops with inedible skins, and heat-processed fruits	Raw consumed plants

In many regions, recycled water is used mainly for agriculture (Table 1), but the same potential cost advantage exists for primarily urban economies, in which industry is a larger sector than agriculture. Al-Zubari (1998) estimates that for Bahrain, secondary treatment of wastewater costs \$0.164/m^3 (\$0.62/tg), tertiary treatment costs \$0.317/m^3 (\$1.20/tg) and desalinated water costs \$0.794/m^3 (\$3.01/tg). If environmental regulations require at minimum secondary treatment for disposal, the additional unit cost (not inclusive of infrastructure expansion costs) of tertiary treatment is relatively small. Thus the marginal cost of a unit of recycled water is likely lower than that of higher quality sources (e.g. groundwater or desalination).

Analyses in the engineering literature have begun to incorporate recycling as an option in large portfolios of water management strategies, but most of these studies do not optimize water use in a truly economic sense. The CALVIN (California value integrated network) model, for example, allocates water statewide within physical, environmental, and selected policy constraints, but its objective is to "maximize the year 2020 net economic benefits of water operations and allocations to agricultural and urban water users" (Jenkins et al., 2001; Draper et al., 2003; Jenkins et al., 2004), not the present value of the stream of net benefits accruing now and in the future as is generally the practice in resource economics. Wilkinson and Groves (2006) also develop a large-scale model whose purpose is to consider the "impact of alternative levels of groundwater conjunctive use and municipal wastewater reuse on long-term supply and demand balance in the region." The model allows a planner to consider the effects of various programs through specification of scenario-specific parameters, i.e. the model does not solve for the optimal economic allocation. Thus, fundamental analytical work on the economics of recycled water remains to be done.

In the absence of recycled water, demand growth necessitates the eventual implementation of a costly but abundant *backstop* resource such as desalination, even if existing water resources are allocated optimally over time to maximize net social benefits (Krulce et al., 1997). The concept of a backstop technology is already established in the groundwater economics literature, even for the case of multiple demand sectors (Koundouri and Christou, 2006). However, little attention has been paid to recycled water and its potential role as an intermediate or sector-specific backstop. Inasmuch as different demand sectors require different qualities of water (e.g. potable vs. non-potable), different resources can serve as backstops for each respective sector.

In developing and solving a dynamic groundwater-economics model to optimize water extraction for two demand sectors, we establish the concepts of an intermediate and a sector-specific backstop. The general model allows for increasing unit recycling costs to implicitly incorporate infrastructure expansion costs for spatially differentiated users. The order of resource extraction for each demand sector optimally follows a least-marginal-opportunity-cost-first rule where the marginal opportunity cost includes extraction, distribution, and endogenous marginal user cost. Recycled water serves as an intermediate resource for non-potable water users in transition to the desalination steady state. We also consider constant unit recycling costs as a special case of the model. In some situations, it may make sense to amortize capital costs to determine a single constant unit cost of recycling. For constant unit recycling cost, recycled wastewater eventually supplies non-potable users as a sector-specific backstop, while desalination supplements groundwater in the household sector steady state. In both cases, recycling water increases welfare by shifting demand away from the aquifer, thus delaying implementation of costly desalination.

2 The model

Groundwater is modeled as a renewable and replaceable resource. Coastal aquifers, often characterized by a "Ghyben-Herzberg" lens (Mink, 1980) of freshwater sitting on an underlying layer of seawater, are "renewable" in that net recharge to the aquifer varies with the groundwater stock. The upper surface of the freshwater lens sits above sea level due to the difference in density between the freshwater and displaced saltwater. The head level (h), or the distance between the top of the lens and mean sea level is a measure of the aquifer stock. Although the stored volume is technically a function of rock porosity, lens geometry and other hydrologic parameters, the head-volume relationship can be approximated as linear (e.g. Krulce et al. 1997; Pitafi and Roumasset, 2009). Thus, as the stock declines, the head level falls proportionately, and groundwater extraction becomes more costly, inasmuch as water must be lifted a longer distance to the surface. Unit groundwater extraction cost is a non-negative, decreasing, convex function of head: $c_G(h_t) \geq 0$, $c'_G(h_t) < 0$, and $c''_G(h_t) \geq 0$.

Leakage from a coastal aquifer is also a function of the head level. In many coastal aquifer systems, low permeability sediment deposits bound the freshwater lens along the coast, but pressure from the lens causes some freshwater to leak or discharge into the ocean as springflow and submarine groundwater discharge. As the head level declines, leakage decreases both because of the smaller surface area along the ocean boundary and because of the decrease in pressure due to the shrinking of the lens. Thus, leakage is a positive, increasing, convex function of head: $L(h_t) \geq 0$, $L'(h_t) > 0$, and $L''(h_t) \geq 0$. Infiltration to the aquifer from precipitation and adjacent water bodies is fixed at a constant rate (I).

Inasmuch as water demand is multifaceted, from a long-term perspective, infrastructure choice should match the varying characteristics of water required for different end-users in terms of quantity and quality. The cost of distributing ground or surface water to users located far from the reservoir or groundwater facility can be non-trivial, but additional infrastructure is only required if new users are beyond the existing network of pipes for potable water conveyance. Non-potable recycled water, on the other hand, requires its own pipes and meters, regardless of the location. Thus, if recycled water users are highly spatially differentiated, infrastructure and distribution costs can quickly escalate with distance from the treatment facility, making recycled water a less cost-effective and hence less desirable resource for distant users.

Properly characterizing the cost of recycled water requires incorporating infrastructure investment into the optimization model. Lumpy investment could be introduced explicitly, but the same general insights can be obtained by assuming that the unit cost of recycling is an increasing and convex function of recycled water, i.e. $c_R(q_t^R) > 0$, $c'_R(q_t^R) > 0$, and $c''_R(q_t^R) \geq 0$. Implicitly, treatment facilities are first constructed near agricultural or industrial centers, i.e. where the concentration of potential recycled water users is highest. The distribution network endogenously expands over time, until eventually it becomes beneficial to build additional treatment plants or to supplement with an alternative resource. For a continuum of non-potable water users, cost increases convexly with units of recycled water because more energy is required to pump water a greater distance, additional treatment facilities may need to be constructed, and costly pipes and meters must be installed for each additional consumer.

Generally, with multiple end-uses or demands and varying qualities of water, users are naturally classified into categories by quality requirements. In some cases, benefits for certain uses may vary by input water quality so that optimality would not always necessitate using the minimum allowable quality for each use. To make the model more tractable and transparent, however, we aggregate non-potable uses into a single demand category (agriculture), and there is no additional benefit to using higher quality water than necessary. Groundwater is the primary source of high quality (potable) water. No surface water is available, but lower quality (non-potable) water can be obtained from wastewater recycling. In addition, desalinated seawater serves as a high quality backstop resource. High quality water can be utilized for both potable and non-potable uses, but recycled water cannot supply the residential/household demand sector.

The production of recycled water is constrained by the quantity of wastewater input and the efficiency of the treatment process. Not all ground or desalinated water used by households enters the sewage system. On average, some proportion, $b_1 \in (0,1)$, is utilized for watering lawns and other outdoor purposes. Of the water that does ultimately enter the sewage treatment facility, a fraction, $b_2 \in (0,1)$, is lost during the treatment process (e.g. as sludge). Consequently, the maximum feasible total production of recycled water in a given period is $\beta(q_t^{GH} + q_t^{BH})$, where $\beta \equiv (1-b_1)(1-b_2)$ is the proportion of groundwater (q_t^{GH}) and desalinated water (q_t^{BH}) used by the household sector that can be effectively recycled.

The water manager chooses the rates of groundwater extraction for the household sector (q_t^{GH}) and the agricultural sector (q_t^{GA}), the rates of desalination for household (q_t^{BH}) and agricultural use (q_t^{BA}), and the rate of wastewater recycling (q_t^{RA}) to maximize the present value of net social benefit, measured as gross consumer surplus less total costs:

$$(1) \qquad \underset{q_t^{GH},q_t^{BH},q_t^{GA},q_t^{RA},q_t^{BA}}{Max} \int_0^{\infty} e^{-\delta t} \left\{ \begin{array}{l} \int_0^{q_t^{GH}+q_t^{BH}} D_H^{-1}(x,t)dx + \int_0^{q_t^{GA}+q_t^{RA}+q_t^{BA}} D_A^{-1}(x,t)dx - \\ (q_t^{GH}+q_t^{GA})c_G(h_t) - (q_t^{BH}+q_t^{BA})c_B - q_t^{RA}c_R(q_t^{RA}) \end{array} \right\} dt$$

$$\text{subject to} \qquad \gamma \dot{h}_t = I - L(h_t) - (q_t^{GH} + q_t^{GA}),$$

$$\beta(q_t^{GH} + q_t^{BH}) - q_t^{RA} \geq 0,$$

where $D_i^{-1}(\bullet)$ is the inverse demand function for sector *i=H,A*, δ is the positive discount rate, c_B is the unit cost of desalinating seawater, and γ is a head-to-volume conversion factor. To incorporate the recycling capacity constraint, we augment the CV Hamiltonian into a Lagrangian function as follows:

$$(2) \qquad \Lambda = \left\{ \begin{array}{l} \int_0^{q_t^{GH}+q_t^{BH}} D_H^{-1}(x,t)dx + \int_0^{q_t^{GA}+q_t^{RA}+q_t^{BA}} D_A^{-1}(x,t)dx - (q_t^{GH}+q_t^{GA})c_G(h_t) - (q_t^{BH}+q_t^{BA})c_B \\ - q_t^{RA}c_R(q_t^{RA}) + \lambda_t[R - L(h_t) - (q_t^{GH}+q_t^{GA})] + \mu_t[\beta(q_t^{GH}+q_t^{BH}) - q_t^{RA}], \end{array} \right.$$

and the Maximum Principle requires that the following conditions hold:

$$(3) \qquad \frac{\partial \Lambda}{\partial q_t^{GH}} = D_H^{-1}(q_t^{GH} + q_t^{BH}, t) - c_G(h_t) - \lambda_t + \beta\mu_t \leq 0 \qquad \text{if} < \text{then } q_t^{GH} = 0$$

$$(4) \qquad \frac{\partial \Lambda}{\partial q_t^{BH}} = D_H^{-1}(q_t^{GH} + q_t^{BH}, t) - c_B + \beta\mu_t \leq 0 \qquad \text{if} < \text{then } q_t^{BH} = 0$$

$$(5) \qquad \frac{\partial \Lambda}{\partial q_t^{GA}} = D_A^{-1}(q_t^{GA} + q_t^{RA} + q_t^{BA}, t) - c_G(h_t) - \lambda_t \leq 0 \qquad \text{if} < \text{then } q_t^{GA} = 0$$

(6) $$\frac{\partial \Lambda}{\partial q_t^{RA}} = D_A^{-1}(q_t^{GA} + q_t^{RA} + q_t^{BA}, t) - c_R(q_t^{RA}) - q_t^{RA} c_R' - \mu_t \leq 0 \qquad \text{if} < \text{then } q_t^{RA} = 0$$

(7) $$\frac{\partial \Lambda}{\partial q_t^{BA}} = D_A^{-1}(q_t^{GA} + q_t^{RA} + q_t^{BA}, t) - c_B \leq 0 \qquad \text{if} < \text{then } q_t^{BA} = 0$$

(8) $$\dot{\lambda}_t - \delta \lambda_t = -\frac{\partial \Lambda}{\partial h_t} = (q_t^{GH} + q_t^{GA}) c_G'(h_t) + \lambda_t L'(h_t).$$

Along the optimal trajectory, the marginal benefit must be equated to the marginal cost of each water resource used in each demand sector. Given our assumption that recycled water is perfectly substitutable for groundwater in the agricultural sector, the marginal benefit, $D_A^{-1}(\bullet)$, is the same, regardless of the source. Similarly, since groundwater and desalinated water are assumed to be perfect substitutes, $D_H^{-1}(\bullet)$ represents the marginal benefit of water in the household sector. The full marginal cost or *marginal opportunity cost* (MOC) of a resource includes not only extraction or treatment cost, but also marginal user cost. The MOC of groundwater and desalinated water for use in the household sector is $\pi_t^{GH} \equiv c_G(h_t) + \lambda_t - \beta\mu_t$ and $\pi_t^{BH} \equiv c_B - \beta\mu_t$ respectively. Similarly, the MOC of groundwater, desalinated water, and recycled water for use in the agricultural sector is $\pi_t^{GA} \equiv c_G(h_t) + \lambda_t$, $\pi_t^{RA} \equiv c_R(q_t^{RA}) + q_t^{RA} c_R' + \mu_t$, and $\pi_t^{BA} \equiv c_B$ respectively. We define the efficiency price for each sector as that which induces the optimal trajectory of water consumption, i.e. the marginal benefit along the optimal paths. For $p_t^H \equiv D_H^{-1}(\bullet)$ and $p_t^A \equiv D_A^{-1}(\bullet)$, conditions (3)-(7) can be simplified to

(9) $$p_t^H = \min\{\pi_t^{GH}, \pi_t^{BH}\}$$

(10) $$p_t^A = \min\{\pi_t^{GA}, \pi_t^{RA}, \pi_t^{BA}\}$$

The price of water for household use is determined by the lower of either the MOC of groundwater or the MOC of desalination. Similarly, the price of water for agricultural use is the minimum of the MOC of groundwater, recycled wastewater, and desalinated seawater. When the recycling capacity constraint is not binding, the MOCs of groundwater are equal in both sectors, as are the MOCs of desalinated water. When the constraint is binding, however, μ_t is the shadow value of an additional unit of recycled water. In optimality conditions (3) and (4), $\beta\mu_t$ is subtracted from the costs because using an additional unit of ground or desalinated water relaxes the recycling constraint and adds to the PV by exactly that amount. On the other hand, μ_t is added to the costs for condition (6) because more recycled water would be used in the absence of the binding constraint. Although the MOC of desalination is constant in both sectors aside from the recycling constraint, the MOCs of groundwater and recycled water are variable. In particular, unit groundwater extraction cost rises as the head level declines, and marginal user cost rises as the resource becomes scarcer. For the reasons previously discusses, unit recycling cost varies with the quantity recycled.

2.1 Steady state

Inasmuch as demand growth ensures desalination in the steady state for both sectors, $p_T^H = c_B - \beta\mu_T$ as per condition (4). That the steady state requires $\dot{h} = 0$ means groundwater extraction for the household sector must be positive, and combining condition (3) with the previous result yields $\lambda_T = c_B - c_G(h_T)$. Taking $\dot{\lambda} = 0$ and plugging

λ_T into condition (8) results in a single equation that can be solved for the unique[1] steady state aquifer head level (h_T^*):

(11) $$c_B = c_G(h_T) - \frac{c_G'(h_T)[I - L(h_T)]}{\delta + L'(h_T)}.$$

2.2 Order of resource use

In this section, we consider three possible scenarios: (a) the unit cost of desalination exceeds the unit cost of recycled water, which exceeds the initial MOC of groundwater; (b) the unit cost of desalination is greater than the initial MOC of groundwater, which is greater than the unit cost of recycled water; and (c) the initial MOC of groundwater is the highest, followed by the unit cost of desalination and then the unit cost of recycled water.

If the aquifer starts in a relatively pristine state, then the initial MOC of groundwater might lie below the cost of the first unit of recycled water. In that case, groundwater optimally supplies both demand sectors in the initial stage of extraction. The MOC of groundwater rises rapidly over time until it reaches the cost of the first unit of recycled water. Groundwater continues to supply both sectors, but as the MOC of groundwater continues to rise, more of the water consumed by the agricultural sector is supplied by recycling, i.e. the network of recycling infrastructure is endogenously expanded as more users optimally switch to the lower quality source. In the steady state, all water resources are used; recycled water is used for the agricultural sector, and desalinated water and groundwater are used for both sectors.

If instead the unit desalination cost exceeds the initial MOC of groundwater, and the initial MOC of groundwater exceeds the unit cost of recycled water for the first unit,

[1] See the appendix for a proof of this result.

then groundwater is extracted exclusively for the household sector and at least some water is recycled for the agricultural sector from the outset. Recycled water is used exclusively for the agricultural sector if the MOC of recycled water at the demand curve is below the MOC of groundwater (and the quantity constraint is not binding). If not, recycled water is used until the MOC of the last unit is just equal to the MOC of groundwater, and groundwater is extracted to satisfy the remainder of the quantity demanded. As the MOC of groundwater rises, more of the agricultural sector's demand is met by recycled water. In the steady state, all water resources are used.

A third possibility is that the aquifer is severely depleted such that the MOC of groundwater starts above the unit cost of desalination. Recycled water is used exclusively by the agricultural sector, unless the quantity constraint is binding or the MOC exceeds the unit cost of desalination at the demand curve, in which case recycling is supplemented by desalination. Desalination is used exclusively by the household sector. The aquifer is allowed to build until the MOC of groundwater falls to the unit cost of the backstop, at which point groundwater and desalination are used simultaneously. In the mean time, the number of recycled users steadily increases until the steady state. Further demand growth in the agricultural sector necessitates eventual supplementation by desalination. The stages of resource use for each scenario are summarized in table 2.

Table 2. Stages of Resource Use

Scenario\Stage	1	2	3
a	GW for H GW for A	GW for H GW + RW for A	GW + DW for H GW + RW + DW for A
b	GW for H RW (+ GW) for A	GW + DW for H GW + RW + DW for A	
c	DW for H RW (+ DW) for A	GW + DW for H GW + RW + DW for A	

Note: GW = groundwater, DW = desalinated water, RW = recycled water; H = household, A = agriculture.

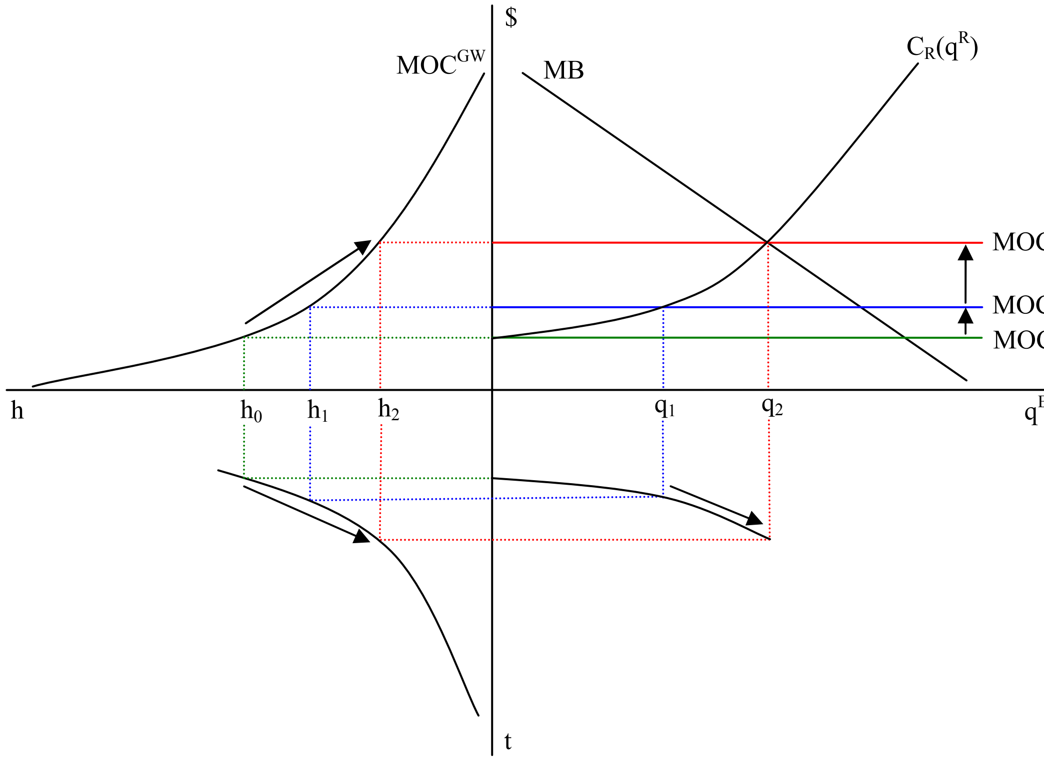

Figure 1. Network of recycled water users expands over time in the agricultural sector.

When water recycling is incorporated into an optimal groundwater management plan, the boundary of recycled water users shifts out over time as the scarcity value of groundwater increases (figure 1).[2] Although the approach path may be non-monotonic, the aquifer head level is eventually drawn down toward its steady state level (SW quadrant of figure 1). As the aquifer is depleted, groundwater becomes scarcer, and its MOC shifts upward (NW quadrant). Given the choice between groundwater and recycled water for the agricultural sector, the source with the lowest MOC is used first. For the head level h_1, and the corresponding groundwater MOC_1, the optimal quantity of recycled water is q_1 (NE quadrant). Up until that quantity, the unit cost of recycled water is lower than the MOC of groundwater, i.e. $C_R(q^R) < MOC_1$. The remainder of the quantity

[2] To maintain graphical clarity, the demand curve is depicted as constant over time. Growing demand does not change the qualitative result that the network of recycled users expands over time.

demanded is met by groundwater at unit cost MOC_1. In later periods, the MOC of groundwater is even higher, which means more recycled water is used, and the boundary of recycled water users expands over time (SE quadrant). Eventually, the system reaches a steady state, at which time expansion ceases and recycling infrastructure is sustained, while the remainder of consumption is met by desalination.

Another way to depict the stages of optimal resource use is to compare directly the time path of each resource's MOC for each demand sector. We again illustrate the optimal program for *scenario a* because it is the most complex of the three. The hypothetical time paths for the other two scenarios can be constructed in a similar manner. For $\pi_0^{GA} < \pi_0^{RA} < \pi_0^{BA}$ and $\pi_0^{GH} < \pi_0^{BH}$, groundwater is used initially in both sectors (figure 2a and 2b). As groundwater scarcity rises, it eventually becomes optimal to use recycled water in the agricultural sector, i.e. $\pi_\tau^{GA} = \pi_0^{RA}$. Meanwhile in the household sector, the scarcity value of groundwater "kinks" slightly because recycling in the agricultural sector lowers groundwater extraction costs by conserving on freshwater. Eventually, the MOC of groundwater and that of recycled water both rise to the MOC of desalination, and the system reaches a steady state. The qualitative welfare implications of the optimal recycling program are revealed when comparing the MOC trajectories to those that would obtain under groundwater optimization alone (figure 2c and 2d). Without recycling, groundwater is used by both sectors until the steady state. Consequently, extraction costs rise more rapidly, as does groundwater scarcity, meaning desalination must be implemented earlier in both sectors. Clearly, implementation of optimal wastewater recycling increases the present value net benefit to society, inasmuch as the lower extraction path allows for an extended period of drawdown before implementation of costly desalination in the steady state.

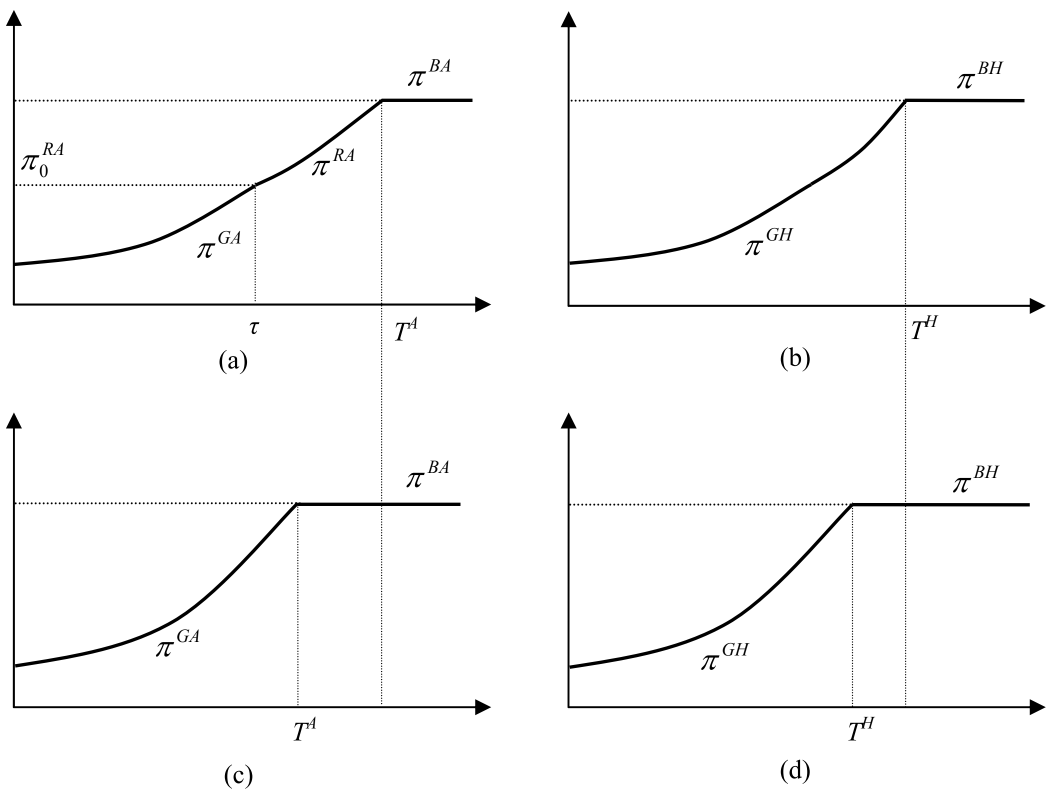

Figure 2. Hypothetical time paths of MOCs: (a) Agricultural sector with recycling, (b) Household sector with recycling, (c) Agricultural sector without recycling, (d) Household sector without recycling.

2.3 Proposed solution method

In the previous section, we discuss the ordering of water resource use as if we already know the optimized MOC paths for each resource within each demand sector. However, solving the problem in practice involves consideration of multiple trial MOC paths, only one of which maximizes PV. It is useful to think of the problem in discrete time for the purpose of computation. If the quantity constraint on recycling is never binding, then the solution method based on the discrete-time analogues of equations (3)-

(10) is fairly straightforward. A trial value is assumed for the initial shadow price of groundwater, and condition (8) allows one to determine the shadow price in the following period. The efficiency price for each sector can then be ascertained from equations (9) and (10) for the current period. The price reveals the current-period rates of extraction, recycling, or desalination. The equation of motion for the aquifer generates the head level for the next period, and the whole process can be repeated, starting with the next period shadow price and head level. Eventually, one of the terminal conditions is reached; either the head level declines to h_T^* or one of the efficiency prices rises to the unit backstop cost. If the conditions do not coincide, i.e. one is inconsistent, the trial value for the initial shadow price of groundwater is revealed as incorrect. The guess must be adjusted and the process repeated until all of the initial and terminal conditions are satisfied for the head level and the efficiency prices in each sector, so that the PV functional is maximized.

If instead the recycling quantity constraint is binding for a finite period, then the computational method should be adjusted. Starting again with condition (8), a trial value for the shadow price of groundwater allows one to solve for the shadow price in the following period. Inasmuch as condition (5) does not depend on the Lagrangian multiplier, one can then determine the efficiency price in the agricultural sector. The price determines the quantity of recycled water, and the quantity of groundwater for the agricultural sector is just the residual of the total quantity demanded at that price. If groundwater is being used in the agricultural sector, it must also be used in the household sector. Condition (3) can be used to solve for the quantity of groundwater in the household sector for $\mu_t = 0$. One can then test the quantity constraint, i.e. check that $\beta(q_t^{GH} + q_t^{BH}) > q_t^{RA}$. If the constraint is not binding, then the aquifer equation of motion generates the head level for the next period. If it is binding, then $q_t^{RA} = \beta(q_t^{GH} + q_t^{BH})$.

Since that does not change the efficiency price in the agricultural sector, condition (6) allows one to determine the value of the Lagrangian multiplier, and the quantity of groundwater used in agriculture is still the residual of the total quantity demanded at that price. Finally, given the value for $\mu_t > 0$, condition (4) yields optimal rate of groundwater extraction in the household sector.

2.4 A special case: constant unit recycling cost

In the case that lumpy infrastructure investment timing is not as crucial (e.g. when the non-household sector is relatively small and stable, and a single treatment facility's capacity would be sufficient) one could use standard amortization methods to approximate a constant unit cost of recycling (c_R). Since recycled water is of less than potable quality, it is a reasonable assumption that the unit cost of wastewater recycling is less than the unit cost of desalinating seawater, i.e. $c_R < c_B$. Recycled water serves as a sector-specific backstop for the agricultural sector. Groundwater is used in every period for household consumption, but recycled water eventually serves the entire agricultural sector in the steady state. Analogous to the general case with rising unit recycling cost, stages of resource use leading to the steady state are determined by the ordering of the three MOCs in each of the demand sectors. Table 3 summarizes the stages of resource use for the same three scenarios discussed in the general cost case.

Table 3. Stages of Resource Use (Constant Unit Recycling Cost)

Scenario\Stage	1	2	3
a	GW for H GW for A	GW for H RW for A	GW + DW for H RW for A
b	GW for H RW for A	GW + DW for H RW for A	
c	DW for H RW for A	GW + DW for H RW for A	

Note: GW = groundwater, DW = desalinated water, RW = recycled water; H = household, A = agriculture.

3 Conclusion

Efficient management of water resources requires optimization over multiple margins, including the development of supplementary resources. Wastewater recycling can delay the costly implementation of desalination but a question arises regarding composition and timing of the requisite investments. Inasmuch as different demand sectors require different qualities of water, it is natural to think of recycled water as an intermediate resource for those users with low water quality requirements. When unit recycling cost is constant, recycled water serves as a sector-specific backstop. Implementation of the model provides guidance on the appropriate timing and size of backstop and recycling infrastructure.

The necessary conditions derived from the optimal control problem accord with a least-marginal-opportunity-cost-first extraction rule, where the marginal opportunity cost of a particular resource is comprised of its extraction cost and endogenous marginal user cost. Inasmuch as the marginal user cost of groundwater is stock-dependent and thus variable over time and the marginal cost of recycled water is increasing in quantity produced, various stages of extraction are possible, depending on initial values and other parameters in the actual application. For example, groundwater may be used exclusively in all sectors for a finite period of time, or it may be that recycled water or desalinated water optimally supplements groundwater in any given period. Although recycled water can never serve as a true backstop for the agricultural sector if demand is growing, it eases the transition of usage from groundwater to desalinated water. More specifically, it increases the present value to society by allowing an extended period of drawdown before implementation of costly desalination.

Water quality is an aspect of the model that should be expanded on in future research. The current model only differentiates between potable and non-potable water, but in reality, there exists many levels of treated water for non-potable uses. While the lowest quality recycled water is acceptable for uses such as industrial cooling, water used for human crops generally requires at least secondary treatment. It remains to be seen whether introducing more finely differentiated categories of end-uses as well as multiple qualities would change the qualitative results presented here.

Appendix

The steady state condition relating price and aquifer head is:

$$c_B = c_G(h_T) - \frac{c_G'(h_T)[I - L(h_T)]}{\delta + L'(h_T)}.$$

Since the unit cost of desalination is constant, the head level that solves the steady state condition is unique if the derivative of the right hand side with respect to h is negative. Applying the quotient rule and the chain rule, differentiating the term yields the following result:

$$c_G'(h) - \frac{[\delta + L'(h)]\{-c_G'(h)L'(h) + c_G''(h)[I - L(h)]\} - c_G'(h)[I - L(h)]L''(h)}{[\delta + L'(h_T)]^2} < 0 .$$

That the term is negative follows from the assumed characteristics of the leakage and extraction cost functions.

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